12.2. Harmonics

Standing Waves

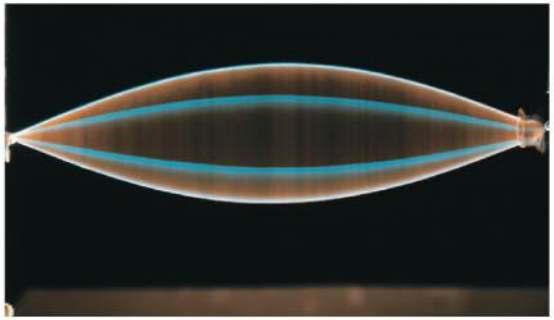
In general, the interference between random traveling waves and their reflected waves produces a time-dependent jumble.

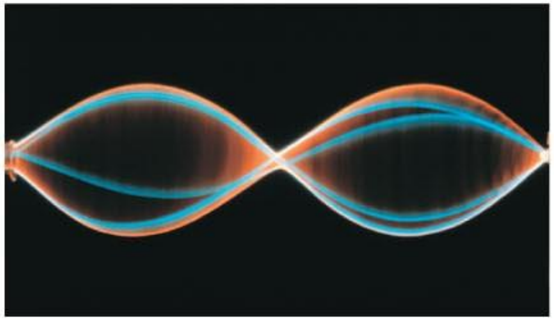
However, when the wave frequency (i.e. λ) at a **resonant frequency**---which is related to the object's length---the reflected waves interfere constructively with the incoming waves and a **standing wave** is formed.

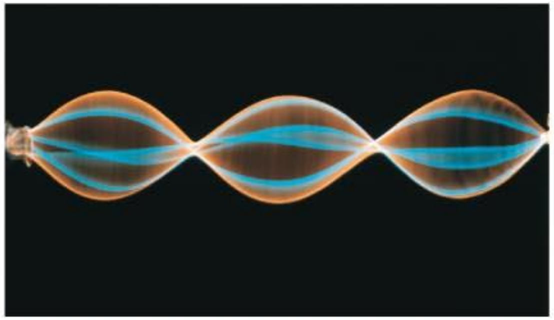
Standing Waves

“Standing” because the wave crests are not moving horizontally.

A **standing wave** is a wave that oscillates in time between fixed end points. This occurs when the total length of the medium is a multiple of half the wavelength---the reflected wave will have antisymmetry with the incoming wave. Thus for a medium of any length, there are certain wavelengths for which standing waves occur.

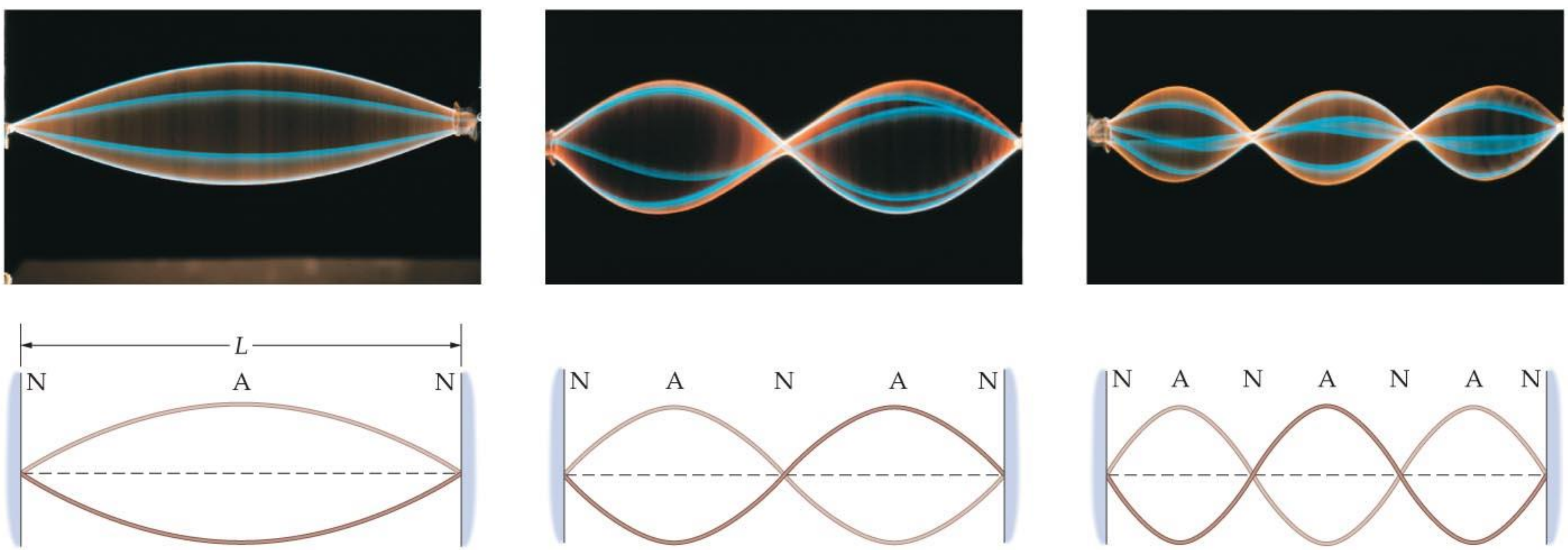






Standing Waves

Points on a standing wave that do not move are called **nodes**. Any end point with a fixed position must be a node (e.g. a guitar string).



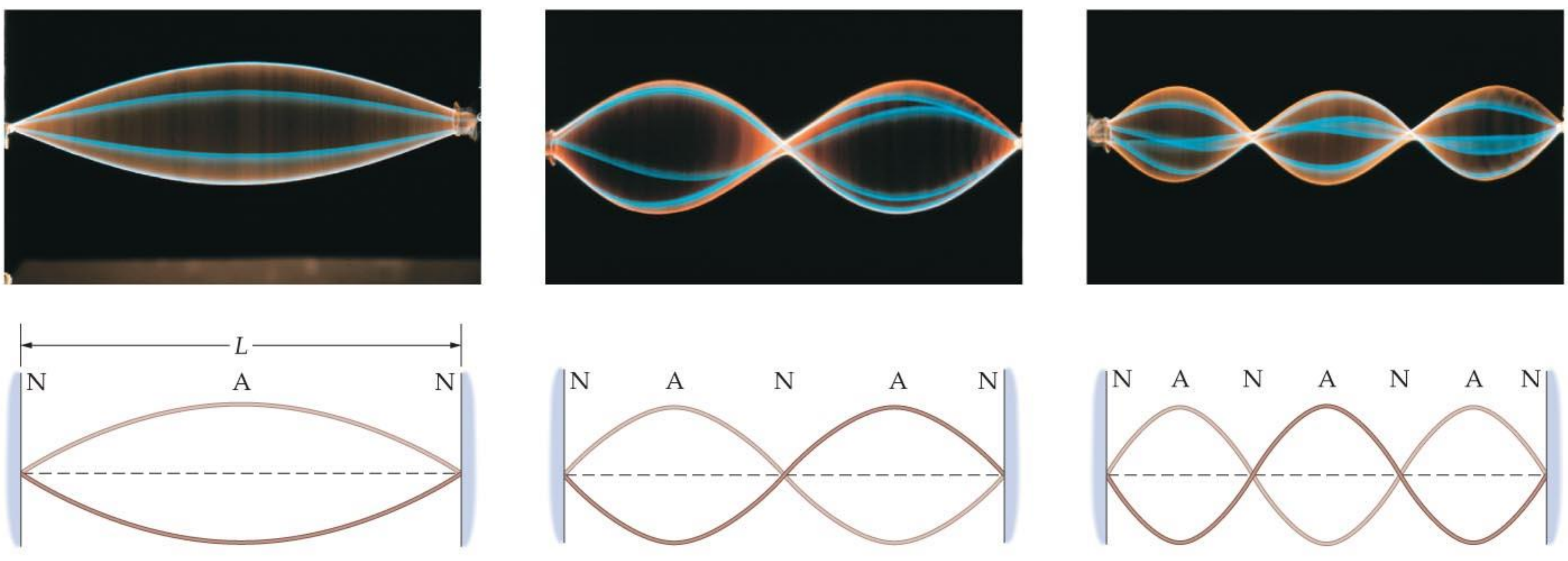
(a) First harmonic (fundamental)

(b) Second harmonic

(c) Third harmonic

Standing Waves

Halfway between two nodes is a point called an **antinode**, where the displacement varies between its positive and negative maximum amplitude. The **fundamental mode (first harmonic)** has one antinode.



(a) First harmonic (fundamental) (b) Second harmonic (c) Third harmonic

Harmonics

A given string has an infinite number of standing wave modes (**harmonics**). The wavelength and frequency can be computed as follows:

First harmonic: $L = \frac{\lambda}{2} \rightarrow \lambda = 2L, f = \frac{v}{\lambda} = \frac{v}{2L}$

2nd harmonic: $L = \lambda \rightarrow \lambda = L, f = \frac{v}{\lambda} = \frac{v}{L}$

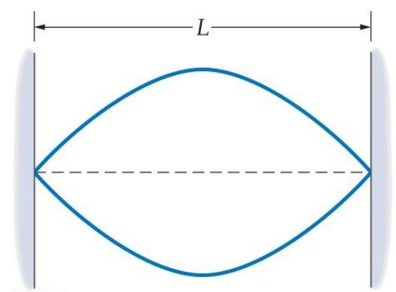
3rd harmonic: $L = \frac{3\lambda}{2} \rightarrow \lambda = \frac{2L}{3}, f = \frac{v}{\lambda} = \frac{3v}{2L}$

…

nth harmonic: $L = \frac{n\lambda}{2} \rightarrow \lambda_n = \frac{2L}{n}, f_n = \frac{v}{\lambda} = \frac{nv}{2L}$

Example 1

A string 1.30 m in length is oscillating in its first harmonic mode. The frequency of oscillation is 7.80 Hz. (a) What is the wavelength of the first harmonic? (b) what is the speed of waves on this string?



$$\lambda = \frac{2L}{n} = 2\mathrm{L} = 2(1.30\ \mathrm{m}) = 2.60\ \mathrm{m}$$

$$v = \lambda f = (2.60\ \mathrm{m})(7.80\ \mathrm{Hz}) = 20.3\ \ \mathrm{m/s}$$

Pitch

As the speed of sound (343 m/s in air) is the same for all frequencies, different frequencies will have different wavelengths according to:

$$v = \lambda f$$

The **pitch** of a sound is the frequency of its sound wave. High notes have a high frequency. One octave equals a doubling of frequency.

Table 14.3 Chromatic Musical Scale

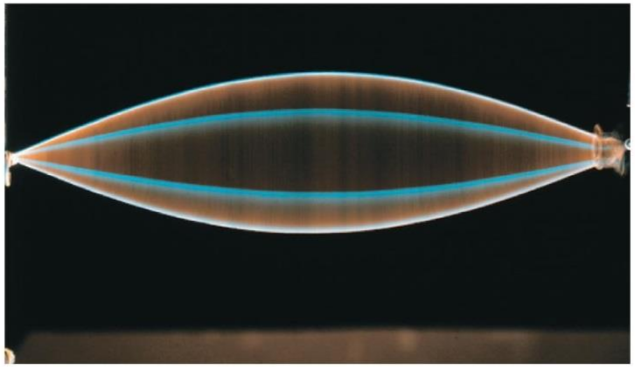
Note	Frequency (Hz)
Middle C	261.7
$C^{\#}$ (C-sharp), $D^{\flat}$ (D-flat)	277.2
D	293.7
$D^{\#}$, $E^{\flat}$	311.2
E	329.7
F	349.2
$F^{\#}$, $G^{\flat}$	370.0
G	392.0
$G^{\#}$, $A^{\flat}$	415.3
A	440.0
$A^{\#}$, $B^{\flat}$	466.2
B	493.9
C	523.3

Stringed Instruments

A plucked (e.g. guitar) string vibrates primarily at its first harmonic. Because

$$f_1 = \frac{v}{2L}$$

a longer string is needed to play a lower frequency (lower pitch) note.

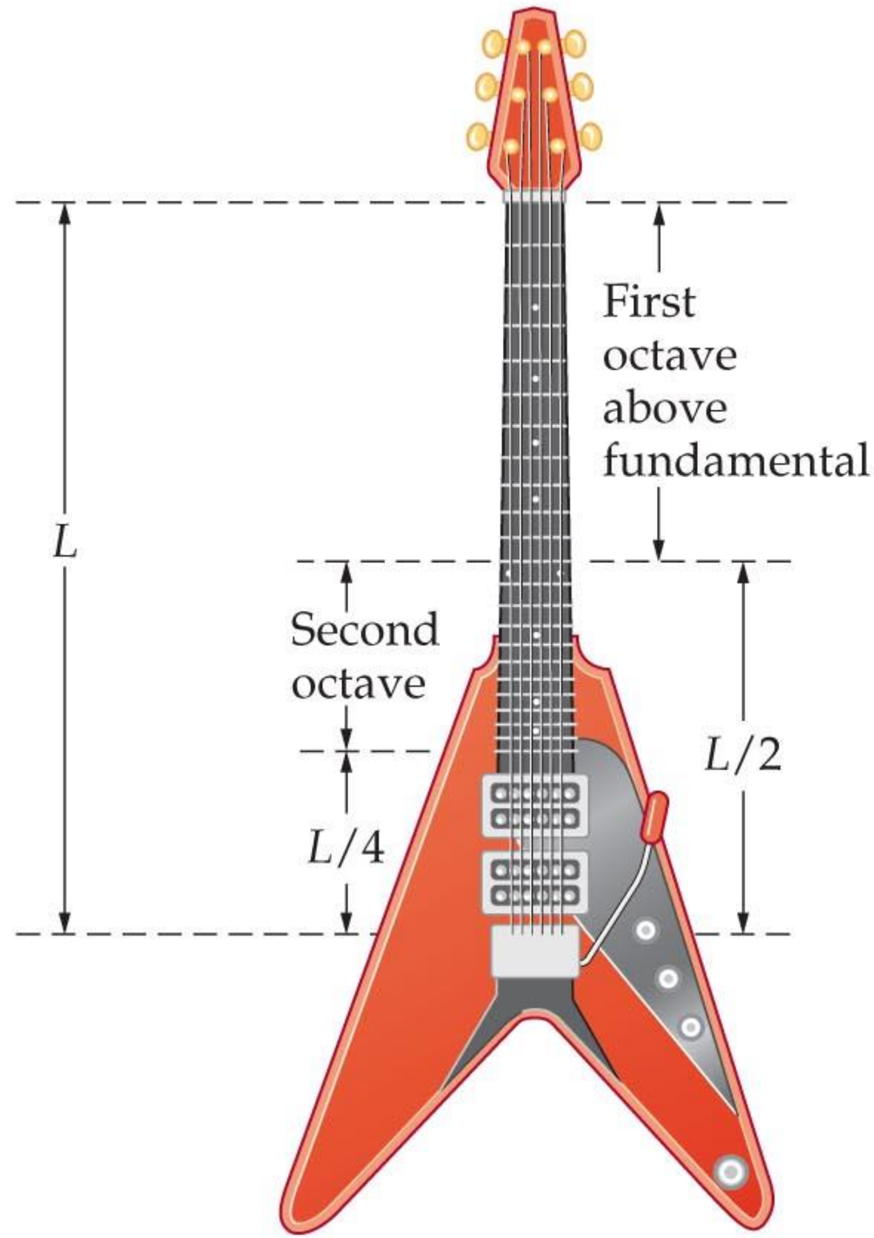


Stringed Instruments

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F_T}{m/L}}$$

In addition to length, string tension and mass per unit length are adjusted to get the right pitch.

Note that the frets are not equally spaced; halving the length raises the pitch one octave.



Stringed instrument demo

Standing Waves

Standing waves are produced not only on strings, but on any object that is struck, such as a drum membrane, or an object made of metal or wood. The object will vibrate (**resonate**) at a resonant frequency.

In singing, our muscles change the tension on our vocal chords to change the frequency of vibration.

In a wind instrument, a column of air vibrates as a standing wave.

Standing wave in a pipe with one open end

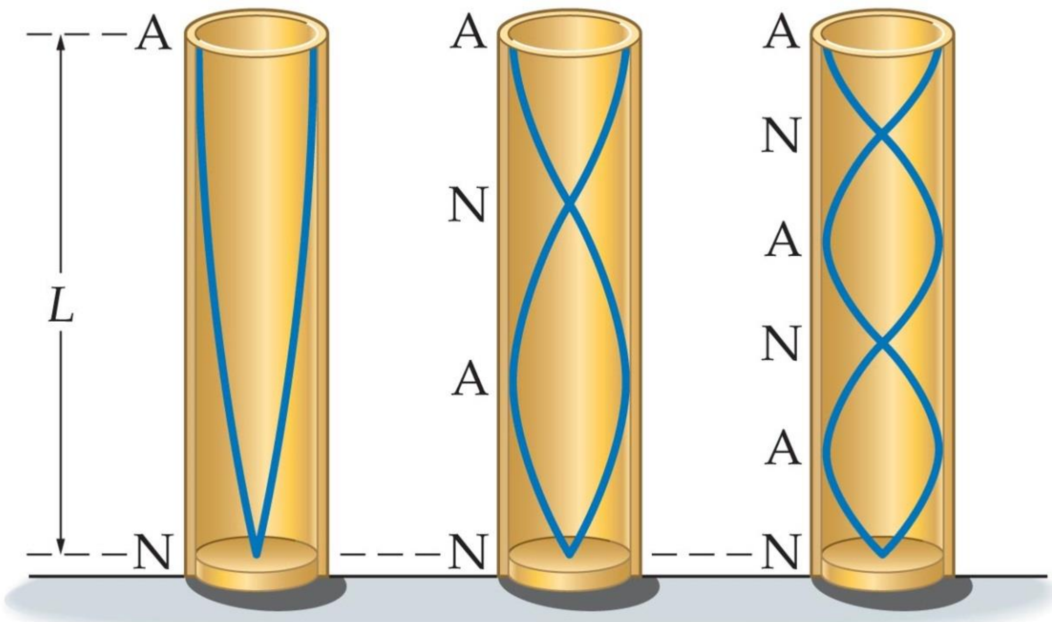
Standing waves can exist in a pipe open at one end. In this case, an antinode is at the open end.

n^{th} harmonic:

$$\lambda_n = \frac{4L}{n}$$

$$f_n = \frac{nv}{4L}$$

for $n = 1, 3, 5, \ldots$



(a) First harmonic (fundamental)

(b) Third harmonic

(c) Fifth harmonic

Standing wave in a pipe with two open ends

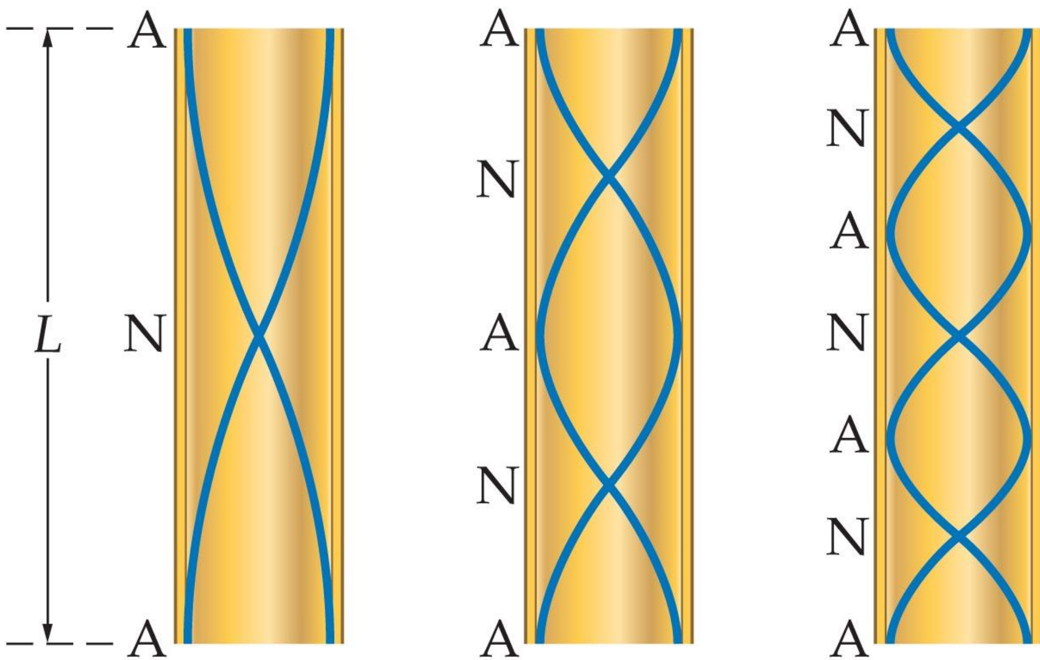
For a pipe with two open ends (like a flute), an antinode is at both ends.

n^{th} harmonic:

$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{nv}{2L}$$

for $n = 1, 2, 3, \ldots$ similar to a string.



(a) First harmonic (fundamental)

(b) Second harmonic

(c) Third harmonic

Harmonics for pipes

For pipes, antinodes occur at the open ends because that is where the air is moving (and varying) the fastest.

Higher harmonics are obtained by "overblowing"---i.e. higher air velocities at the open end.

Interestingly, sharps and flats can be produced by "half-holing".

Example 2

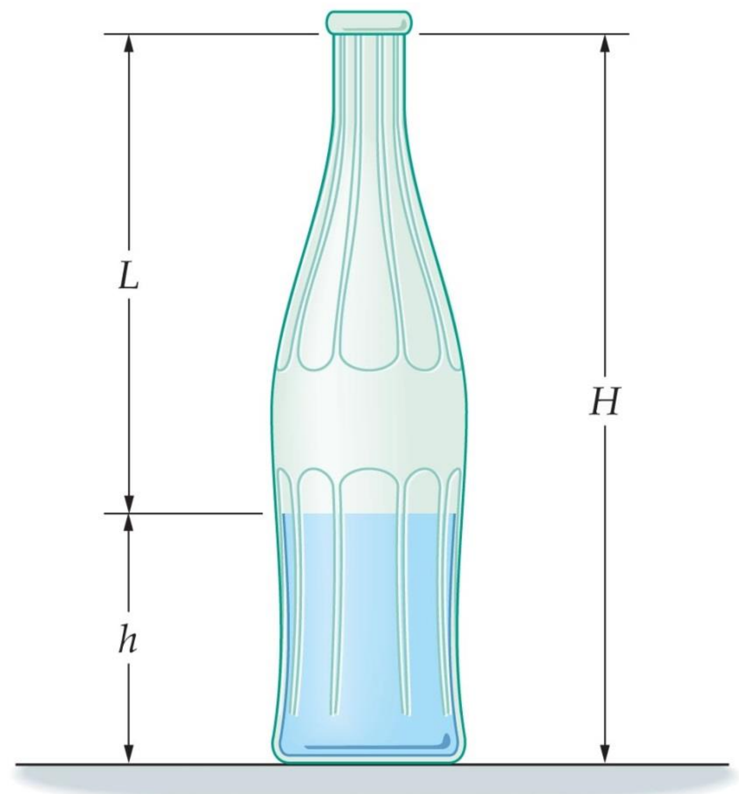
You want to use a soda bottle as a musical instrument. If the bottle is 26.0 cm tall, and you want the first harmonic to be 525 Hz, how high should it be filled with water?

Treat is as a 1-end pipe:

$$f_1 = \frac{1v}{4L} \rightarrow L = \frac{v}{4f}$$

$$L = \frac{343\ \text{m/s}}{4(525\ \text{Hz})} = 0.163\ \text{m}$$

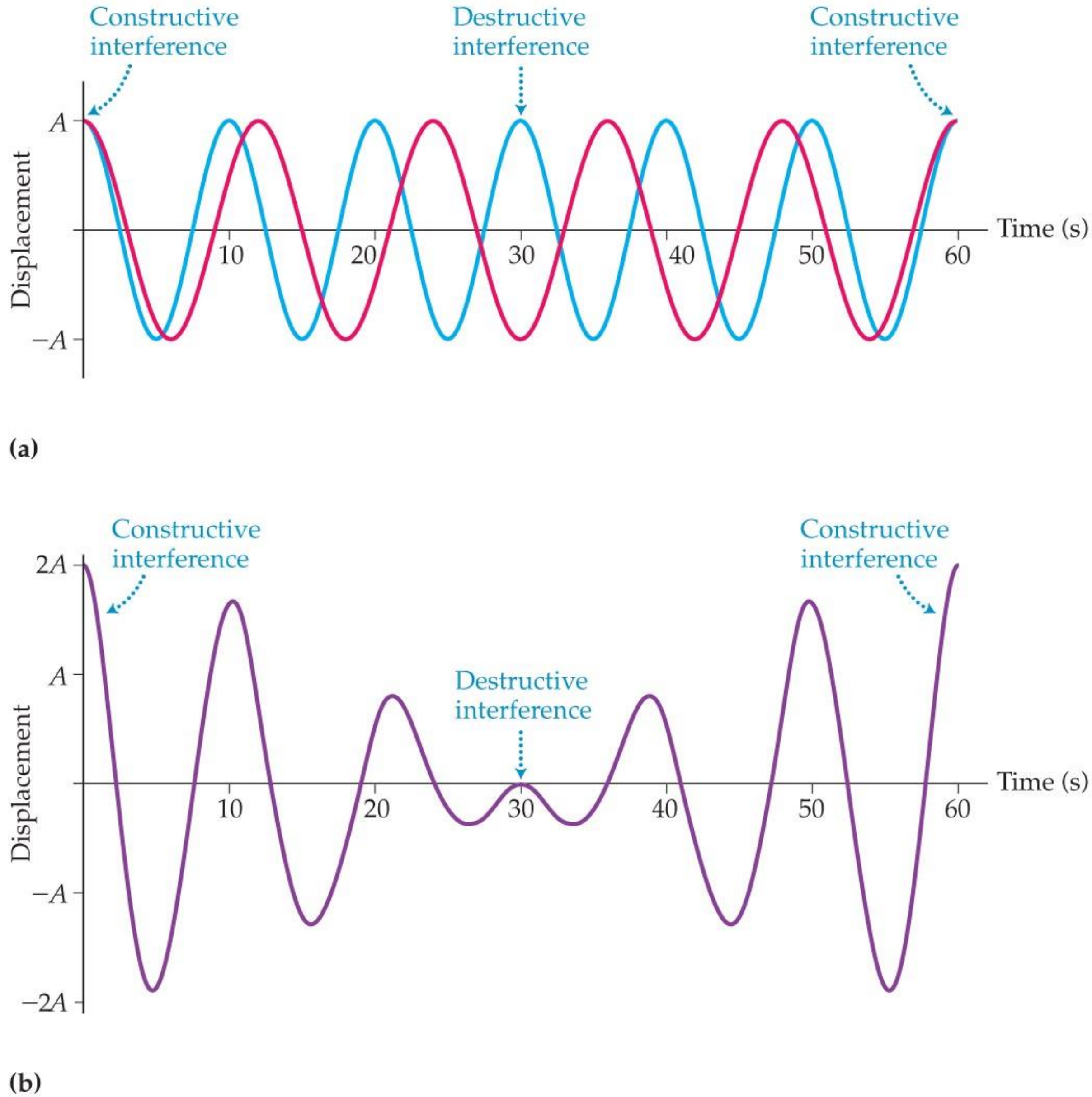
$$h = H - L = 9.7\ \text{cm}$$



Sounding Board/Box

Strings are too thin to compress much air. Therefore stringed instruments typically used sounding boards (piano) or sounding boxes (guitar, violin) which amplify the sound. They too begin to vibrate at the frequency of the string, and can expand and compress more air because of their larger surface area, producing a louder sound.

Beats



Constructive interference
Destructive interference
Constructive interference
A
−A
Displacement
Time (s)
10
20
30
40
50
60
(a)
Constructive interference
Constructive interference
Destructive interference
2A
A
−A
−2A
Displacement
Time (s)
10
20
30
40
50
60
(b)

PhET Simulations

http://phet.colorado.edu/en/simulation/normal-modes

http://phet.colorado.edu/en/simulation/wave-on-a-string